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segment $AD' = AD$ and determine on it the point E so that $\frac{AE}{AD'} = k$. The perpendicular EB to AD' at the point E will meet q in a point B which is the second vertex, besides A, of the required triangle. The third vertex C is easily found.

This construction is applicable whether the given lines are concurrent, parallel, or form a triangle.

To satisfy the conditions of the generalized problem, it is necessary to exercise care in effecting the rotation in the proper sense. If the problem should read that ABC is to be similar to a given triangle without specifying which angle shall have its vertex on which of the given lines, the point A would be the origin of three triangles each solving the problem, but differently situated. The three triangles coincide, if the angles of ABC are to be equal to each other, as in the case of the proposed problem.

The particular case was also solved by Daniel Kreth, Elizabeth B. Davis, Clifford N. Mills, S. A. Joffe, Elmer Schuyler, J. W. Clawson, and the Proposer.

455. Proposed by R. P. BAKER, University of Iowa.

Find the minimum triangle of assigned angles inscribed in a given triangle.

Solution by Roger A. Johnson, Western Reserve University.

By the well-known theorem of Miquel (McClelland, *Geometry of the Circle*, pp. 40–42), if a triangle PQR be inscribed in a given triangle ABC so as to be directly similar, i. e., without being turned over, to a given triangle; then the circles AQR, BRP, CPQ pass through a point O, which, moreover, is fixed for all positions of the inscribed triangle; further, if P, Q, R lie on the sides of the triangle, and not on their extensions, $\angle OPB = \angle OQC = \angle ORA$. If from the point O, lines OP_1, OQ_1, OR_1 be drawn, making equal angles with the sides, their extremities P, Q, R, will form a triangle similar to PQR.

It follows that if the point O be known, we shall obtain the minimum triangle by taking the pedal triangle; that is to say, the triangle whose vertices are the feet of the perpendiculars from O to the sides of ABC.

First inscribe in ABC any triangle of the given species. The following, due to Petersen, is, perhaps, the simplest. Take Y on AC and Z on AB, at random. On YZ construct a triangle XYZ of the given type and in the desired position. Let AX cut BC at P. Through P draw PQ and PR parallel to XY and XZ, respectively. Then QR will be parallel to YZ and PQR will be an inscribed triangle of the desired type.

Now draw the circles AQR, BRP, CPQ, intersecting again at O. From O, drop perpendiculars OL, OM, ON, to the sides. Then LMN is the required triangle.

Also solved by C. N. Schmall, Frank Irwin, and J. W. Clawson.

CALCULUS.

368. Proposed by PAUL CAPRON, Annapolis, Maryland.

Develop $\log_{10} x/\sin x$ and $\log_{10} \tan x/x$ to three terms as functions of $\log_{10} \sec x$, showing that

if x is less than $7° 15'$, then to five decimals, $\log_{10} x = \log_{10} \sin x + \frac{1}{3} \log_{10} \sec x = \log_{10} \tan x - \frac{2}{3} \log_{10} \sec x$.

Solution by the Proposer.

$$y = \log \frac{x}{\sin x}, \quad \text{or} \quad e^y = \frac{x}{\sin x}; \quad \frac{dy}{dx} = \frac{1}{x} - \cot x.$$

$$z = \log \sec x, \quad \text{or} \quad e^z = \sec x; \quad \frac{dz}{dx} = \tan x.$$

Hence,

$$\frac{dy}{dx} = \tan x \frac{dy}{dz}.$$

Also

$$z - y = \log \frac{\tan x}{x}, \quad \text{or} \quad e^{z-y} = \frac{\tan x}{x}.$$

$$\tan x \frac{dy}{dz} = \frac{1}{x} - \cot x; \quad \tan^2 x \frac{dy}{dz} = \frac{\tan x}{x} - 1, \quad \frac{dy}{dz} = \frac{e^{z-y} - 1}{e^{2z} - 1} = \frac{e^{-y} - e^{-z}}{e^z - e^{-z}}.$$

$$z = 0 = y \quad \text{when} \quad x = 0.$$

$$\left.\frac{dy}{dz}\right]_{z=0} = \frac{0}{0} = \left.\frac{-\left.\frac{dy}{dz}\right] e^{-y} + e^{-z}}{e^z + e^{-z}}\right]_{z=0}; \quad \left.\frac{dy}{dz}\right]_{z=0} = \frac{-\left.\frac{dy}{dz}\right]_{z=0} + 1}{2}; \quad \left.\frac{dy}{dz}\right]_{z=0} = \frac{1}{3}.$$

$$\left.\frac{d^2y}{dz^2}\right]_{z=0} = \frac{1}{(e^z + e^{-z})^2}\left[2 - e^{-y}\left([e^z - e^{-z}]\frac{dy}{dz} + [e^z + e^{-z}]\right)\right]; \quad \left.\frac{d^2y}{dz^2}\right]_{z=0} = \frac{0}{0}.$$

$$\left.\frac{d^2y}{dz^2}\right]_{z=0} = \frac{1}{2(e^{2z} - e^{-2z})}\left[- e^{-y}\left[(e^z - e^{-z})\frac{d^2y}{dz^2} - (e^z - e^{-z})\left(\frac{dy}{dz}\right)^2 + (e^z - e^{-z})\right]\right]_{z=0}$$

$$= \frac{- e^{-y}}{2(e^z + e^{-z})}\left[\frac{d^2y}{dz^2} - \left(\frac{dy}{dz}\right)^2 + 1\right]_{z=0}; \quad \left.\frac{d^2y}{dz^2}\right]_{z=0} = -\frac{8}{45}.$$

$$y = \frac{z}{3} - \frac{8}{45}z^2 \cdots, \quad z - y = \frac{2}{3}z + \frac{8}{45}z^2 \cdots,$$

or

$$\log x - \log \sin x = \frac{1}{3}\log \sec x - \frac{8}{45}(\log \sec x)^2,$$

$$\log_{10} x - \log_{10} \sin x = \frac{1}{3}\log \sec x - \frac{8}{45M}(\log \sec x)^2.$$

Similarly,

$$\log_{10} \tan x - \log_{10} x = \frac{2}{3}\log \sec x + \frac{8}{45M}(\log \sec x)^2,$$

$$\frac{8}{45M}(\log_{10} \sec x)^2 = .000005 \text{ if } \log_{10} \sec x = .0034948, \; x = 7° \, 15',$$

$$\frac{8}{45M} = .4 \text{ nearly}, \quad = (.40926), \quad = \frac{70}{171} \text{ very closely}.$$

This theorem enables one to dispense without inconvenience with the usual table for small angles. It is well to take $\log_{10}$ c.m. $1' = 6.46372\frac{2}{3} - 10$ in order to account for odd thirds. The theorem gives correct results in most instances for values of θ up to about $10°$. For $\theta = 10°$ there is an error of one unit in the fifth place of decimals.

369. Proposed by I. A. BARNETT, Chicago, Ill.

Compute the definite integral $\int_a^b \log x dx$ by direct summation.